Measuring disease and exposure - Assignment solutions

1. "b" & “c” are correct; shorter duration can lower prevalence despite rising incidence. “a” is incorrect, as the prevalence would increase, not decrease, with increasing chronicity. “d” is incorrect, as prevention should reduce the incidence.

2. 

3. 

   a. 0.125 (1 case with 8 persons at risk)

   \[
   \text{Prevalence} = \frac{\text{Cases present in a population at a specified time}}{\text{Number of persons in that population at that time}}
   \]

   b. 0.250 (2 cases with 8 persons at risk)

   c. person days at risk = 689:

   Total person days = 91 days (3 mos.) x 8 persons = 728.

   There are 39 days within this 3-month period when individuals are not at risk because they are already ill (B loses 12 days within the period of observation 9/1 - 11/30 inclusive, C loses 10 days, F loses 5 days, and G loses 12 days): 728 - 39 = 689 person-days

   d. Incidence density:

   \[
   \text{Average incidence density} = \frac{\text{Number of new cases}}{\text{Population time at risk}} = \frac{5}{689} = 0.0073 \text{ cases per person-day}
   \]
Specification of units for incidence density is essential, since the number has no meaning in itself (for example, the incidence density could be expressed per person-week, per person-month, etc., with a different numerical value for the incidence density in each case). In contrast, proportions have no units, though a scaling factor is often used in order to write the number in a more readable fashion, e.g., 153 per 100,000 is a more easily read number than 0.00053, but either form is correct and complete for prevalence or incidence proportion.

4.

a. Rate (relative)
b. Proportion
c. Proportion
d. Neither - this is (only) a ratio
e. Rate (relative) - change in cases / change in time relative to population
5. a. Flow Diagram

50 Alive (Incident cases)

100 Develop Q

50 Died (Incident cases)

1,000 without Q

135 Died, without Q (Censored)

765 Alive, well (Unaffected)

50 with Q

10 Alive, cases (Prevalent cases)

40 Died, with disease (Deceased cases)

Population of size 100,000
Random sample of size 1,050

f. (i) point prevalence at the initial examination:

50/1050 = .048, or 48 cases per thousand

(ii) 5-year cumulative incidence:

\[
\text{Cumulative incidence} = \frac{\text{Number of new cases}}{\text{Population at risk}}
\]

There were 100 new cases and 1000 disease-free persons at the start of the period. Therefore:
CI = \frac{100}{1,000} = 0.10, or 100 per 1,000

However, 135 persons died of other causes than X and therefore were not actually “at risk” of developing disease Q, at least not throughout the 5 years. Omitting them gives:

CI = \frac{100}{865} = 0.116, or 116 per 1,000

The former CI (0.10) probably underestimates the “true” CI, since it implicitly assumes that none of the 135 persons who died of other causes would have developed disease Q had he lived. The latter CI may overestimate the “true” CI since, after all, the 135 who died were available to get disease Q and be detected during part of the follow-up period.

A compromise solution is to estimate the CI by taking into account the follow-up time on those subjects who died of other causes (or who withdrew from the study for other reasons). One method is:

CI = \frac{Q}{(N - W/2) (1,000 - 135/2)} = \frac{100}{(1,000 - 135/2)} = 0.107

Where:  
Q = new cases of disease Q  
N = initial cohort (disease free)  
W = withdrawals

This method assumes that:

- subjects withdrew (died) evenly throughout the period (i.e., that they withdrew, on the average, at the midpoint).
- subjects were in fact at risk of disease (and detection of disease) prior to withdrawal - e.g., if they had developed disease Q, it would have been noted at the time of their death.

If the loss to follow-up is small, the results of each method will be about the same. An intensive search for a random sample of those originally lost to follow-up can be invaluable in assessing bias.

(iii) Average incidence density
New cases $Q$ = \[ \frac{\text{ID}}{\text{Population time at risk}} = \frac{Q}{\frac{1}{2}(N_1 + N_0)(\Delta t)} \]

Where: $Q$ = new cases

$N_1$ = size of initial cohort

$N_0$ = number alive and well at follow-up

$\Delta t$ = length of follow-up

So that:

\[ \frac{100}{\frac{1}{2}(1000 + 765)(5)} = 0.023/\text{year} = 23 \text{ cases per 1,000 py} \]

The same result can be obtained from:

\[ \frac{Q}{\frac{1}{2}(N_1 + N_0)(\Delta t)} = \frac{100}{(1000 - \frac{1}{2}[100] - \frac{1}{2}[135])(5)} \]

(iv) 5 yr case fatality rate:

\[ \frac{\text{Deaths from } Q}{\text{Cases of } Q \text{ at initial exam}} = \frac{40}{50} = 0.80, \text{ or } 80\% \]

(v) Prevalence of disease at the reexamination (1965):

\[ \frac{60}{825} = 0.073 = 73 \text{ cases per 1,000} \]

a. The lower and upper limits of proportions are 0 and 1, respectively.

b. Incidence density is an average rate, not a proportion.

c. The assumption is that the distribution of duration of the disease is similar between the two case groups. Information on age, sex, and other potentially relevant characteristics would also be desirable.
d. Cumulative incidence would be used to estimate risk. In probability terms, 
Pr(D | at risk for 5 years)=0.107, or an individual in the study population had a 10.7% chance 
of developing disease Q in the next 5 years if he does not first die of another cause during 
that period.

6. Definitions:

a. Cumulative Incidence - the proportion of new cases that develop in a population at risk of 
getting the disease, over a stated period of time.

b. Incidence Density - the rate at which new cases develop per unit time, relative to the size of 
a population at risk of getting the disease.

c. Prevalence - the number of existing cases of a disease as a proportion of a defined 
population at a specified point in time (or short period of time).

7. The three basic components of incidence are:

a. the number of new cases

b. the population at risk

c. the period of observation or follow-up.

8.

a. Treatment A was superior in prolonging life. Even though the proportion of patients dying 
by year 6 was the same for each treatment, patients receiving treatment A tended to survive 
longer (die later during the follow-up period).

b. The value of a survival ratio would depend upon the (arbitrary) choice of time period. For 
example, in the graph shown, the 3-year survival advantage for treatment A is very small, the 
5-year advantage is quite large. Survivorship analysis considers the time-to-death for patients 
in the two groups, providing a fuller basis for comparison. After all, by the end of a long 
足够的 follow-up period, all subjects will be dead! The aim of medical treatment (and health 
promotion) is, among other things, that we should die later.